

1. (10p) We place 7 balls in two boxes at random. What is the probability that there will be balls in both boxes?  $(1 - 2 \cdot \frac{1}{2^7})$
2. (10p) A box contains 7 red and 4 black balls. We draw three of them without replacement. What is the probability that the first two balls are red and the third is black?  $(\frac{7}{11} \cdot \frac{6}{10} \cdot \frac{4}{9})$
3. (10p) A spam filtering program monitors the frequent words in the email messages. Suppose that 80% of the emails is spam. 10% of the spams contain the word "free" and only 1% of the non-spam letters contain the word "free". Somebody just received an email and it contains the word "free". What is the probability that it is a spam?  $(\frac{0.80 \cdot 0.10 + 0.20 \cdot 0.01}{0.80 \cdot 0.10} = 0.9975)$
4. (10p) A man states that he can see the future. To test his abilities they toss a fair coin ten times and they ask him before every toss to guess the result. He guessed seven times correctly. What would be the probability that he guesses at least seven times correctly if he did not have that stated supernatural ability?  $(\binom{10}{7} \frac{1}{2^{10}} + \binom{10}{8} \frac{1}{2^{10}} + \binom{10}{9} \frac{1}{2^{10}} + \binom{10}{10} \frac{1}{2^{10}} = 0.1718)$
5. (10p) The weight of a random dog in a certain village follows normal distribution with expectation 20kg and standard deviation 7kg. What is the probability that a random dog
  - (a) weighs at least 27kg?  $(1 - \Phi(1) = 1 - 0.8413 = 0.1587)$
  - (b) weighs at least 27kg provided that it weighs at least 20kg?  $(\frac{1 - \Phi(1)}{1 - \Phi(0)} = \frac{0.1587}{0.5000} = 0.3174)$